

- 3) (P1) For the four link mechanism shown in the figure, find:
- 1) The linear accelerations of points B, C, D & E.
 - 2) The angular accelerations of links BC, CD, & EE.

Solution - Draw the configuration diagram to a suitable scale.

Draw the velocity diagram to a suitable scale.

1) $V_{ba} = r \cdot \omega_{ab} = (0.05)(12) = 0.6 \text{ m/s}$

2) Complete the velocity diagram as usual.

For drawing the acceleration diagram, set up a vector table as below -

No.	Vector	Magnitude (m/s^2)	Direction	Sense
1	a_{ba}^t	$\alpha_{AB} \cdot l(AB) = 20(0.05) = 1 \text{ m/s}^2$	\perp to AB	towards in vel. diag. (*)
2	a_{ba}^c	$\frac{V_{ba}^2}{l(AB)} = \frac{(0.6)^2}{(0.05)} = 7.2 \text{ m/s}^2$	\parallel to AB	towards A in config. diag.
3	a_{cb}^c	$\frac{V_{cb}^2}{l(BC)} = \frac{(0.4)^2}{(0.065)} = 2.46 \text{ m/s}^2$	\parallel to BC	towards B in config. diag.
4	a_{cb}^t	$\alpha_{BC} \cdot l(BC) = 1.9 \text{ m/s}^2 \therefore \alpha_{BC} = 29.23 \text{ rad/s}^2$	\perp to BC	—
5	a_{cd}^c	$\frac{V_{cd}^2}{l(CD)} = \frac{(0.465)^2}{(0.055)} = 3.93 \text{ m/s}^2$	\parallel to CD	towards D in config. diagram
6	a_{cd}^t	$\alpha_{CD} \cdot l(CD) = 6.55 \text{ m/s}^2 \therefore \alpha_{CD} = 119.09 \text{ rad/s}^2$	\perp to CD	—

(*) Since α_{AB} is also clockwise, a_{ba}^t is in the direction of V_{ba} . If α_{AB} is anticlockwise (or α_{AB} is retardation), a_{ba}^t will be in opposite direction of V_{ba} .

In short, the tangential component of acceleration is to be shown in the direction of the angular acceleration.

- 1) Draw a_{ba}^c first, since both magnitude and direction are known. Thus a_{ba} is drawn.
- 2) Draw a_{ba}^t . Thus a_{ba} is drawn. The total accⁿ of AB is shown by a_b .
- 3) Draw a_{cb}^c , since both magnitude and direction are known. a_{cb}^t is towards 'c' in the vel. diag. Draw a line \perp to BC through 'c'.
- 4) a_{cd}^c is known in magnitude and direction. Through 'd', draw 'dc' to proper scale to represent a_{cd}^c .
- 5) a_{cd}^t must be \perp to BCD. Through 'd', draw a line \perp to BC. The intersection of this line with the line drawn through 'c' will mark 'c'.

Thus,

Total accⁿ of B relative to A = a_{ba}

Total accⁿ of C relative to A = a_{ca}

Total accⁿ of C relative to B = a_{cb}

From the acceleration diagram,

1) Linear acceleration of B

$$= a_{ba}$$

$$= 7.25 \text{ m/s}^2$$

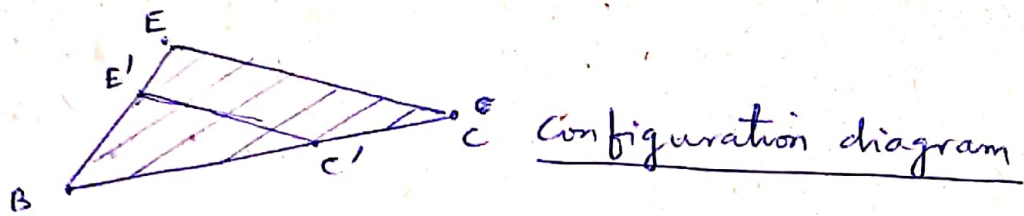
2) Linear acceleration of C = $a_{ca} = 7.6 \text{ m/s}^2$

3) Linear acceleration of E = $(**)$

'E' is an offset point.

To find the acceleration of an offset point, on a link, a triangle similar to the one formed in the configuration diagram, can be made on the acceleration image of the link in such a way that the sequence of letters is the same, i.e. BEC is clockwise, so should be bfc on the acceleration diagram.

a)



An easier method of making triangle 'bec' on the accⁿ diagram similar to BFC on the config. diagram is by marking BC' on BC equal to bc' (on accⁿ diag.) and drawing a line C'E' || to CE. BE'C' is the exact size of Δ to be made in the accⁿ diag.

Take $be = BE'$
(accⁿ diag.) (config. diag.)

& $ce = C'E'$
(accⁿ diag.) (config. diag.)

Thus, point 'e' can be obtained on accⁿ diagram.

Linear accⁿ of E = $a_{ea} = 7.8 \text{ m/s}^2$

4) Angular acceleration of link BC -

From the vector table, it can be seen that

$$a_{cb}^t = \alpha_{BC} \cdot l(BC)$$

Hence α_{BC} can be found out if a_{cb}^t is known.

From the accⁿ diagram, $a_{cb}^t = 1.9 \text{ m/s}^2$.

$\therefore \alpha_{BC} = 29.23 \text{ rad/s}^2 \curvearrowright$

{ If we look at point C from point B, point C relative to B will have a tangential accⁿ in the direction of a_{cb}^t . Hence it can be seen that α_{BC} is anticlockwise }

5) Angular acceleration of link CD -

$$a_{cd}^t = \alpha_{CD} \cdot l(CD) = 6.55 \text{ m/s}^2$$

$\therefore \alpha_{CD} = 119.09 \text{ rad/s}^2 \curvearrowright$

If we look at C from D, C relative to D will have a tangential acceleration in the direction of a_{cd}^t . Hence it can be seen that α_{cd} is anticlockwise.

b) Linear acceleration of F

F is an intermediate point on link BC.

The acceleration of an intermediate point on the link can be obtained by dividing the acceleration vector in the same ratio as the point divides the link.

For point F on link BC,

$$\frac{BF}{BC} = \frac{bf}{bc}$$

(config. diag.) (accⁿ diag.)

Thus, point 'f' can be located on the accⁿ diagram.

From the acceleration diagram,

$$a_f = a_{fa} = 7.25 \text{ m/s}^2$$

Vector eqⁿ for accⁿ

1) $a_b = a_{ba} + a_a$
 $\therefore a_b = (a_{ba}^c + a_{ba}^t) + (a_a^c + a_a^t)$

2) $a_c = a_{cb} + a_b$
 $\therefore a_c = (a_{cb}^c + a_{cb}^t) + (a_{ba}^c + a_{ba}^t)$

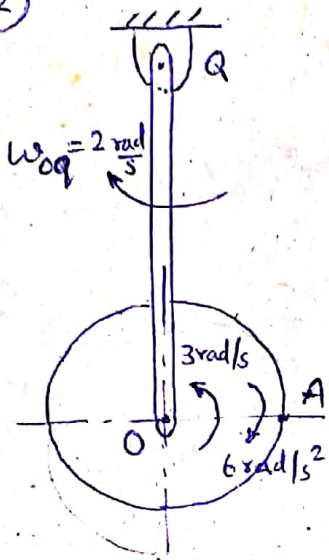
3) $a_e = a_{eb} + a_b$
 $\therefore a_e = (a_{eb}^c + a_{eb}^t) + (a_{ba}^c + a_{ba}^t)$

Note ****** Point E can also be located by considering a_{eb}^c & a_{eb}^t , and a_{ec}^c & a_{ec}^t . The intersection of a_{eb}^t and a_{ec}^t will locate 'e'. (a_{ec}^c & a_{eb}^c are known)

****** Point E can also be located by considering a_{eb}^c & a_{eb}^t . a_{eb}^c is known in direction & magnitude. $a_{eb}^t = \lambda(EB)\alpha_{eb} = \lambda(EB)\alpha_{bc}$ & hence is known in direction & magnitude.

5)

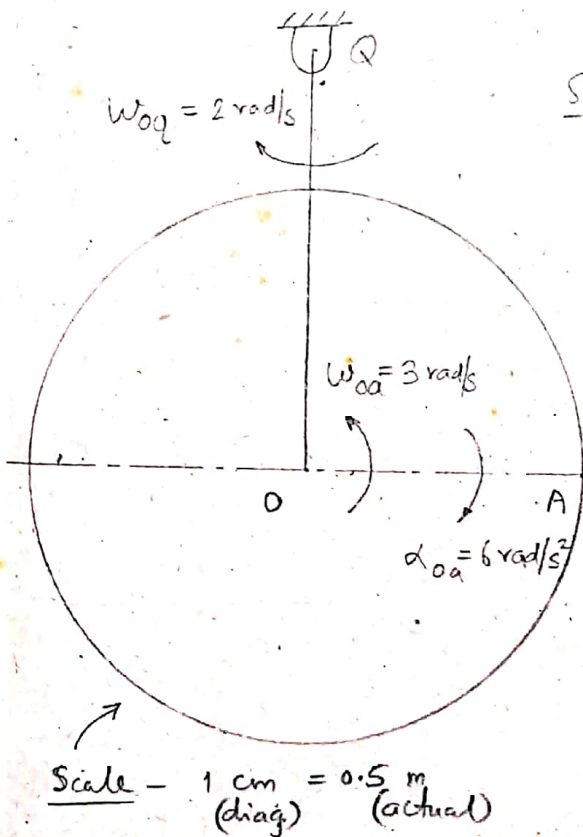
(P2)



Arm OQ of length 3 m , rotates about a fixed point Q with a constant angular speed of 2 rad/s clockwise as shown in the figure. At the same time, the circular disc of radius 2 m , rotates about its center O at an angular speed of 3 rad/s anticlockwise, and at an angular acceleration of 6 rad/s^2

clockwise, both relative to OQ . The two motions occur in the same plane. Find the absolute acceleration, in magnitude and direction, of the point A on the periphery of the disc, given that $\angle AOR = 90^\circ$ at the instant under consideration. (Dadara 16)(Q4A)(8)

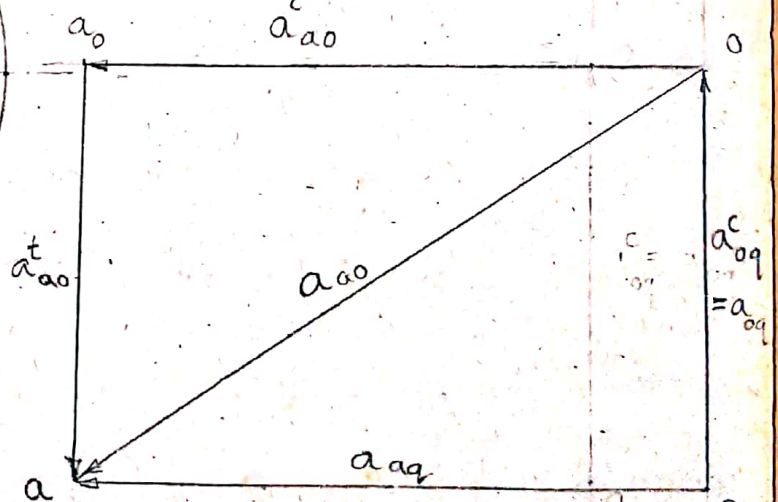
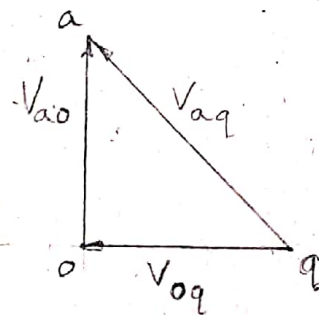
Solution -



Vel. diag.

Scale -

$1\text{ cm} = 2\text{ m/s}$
(diag) (actual)



Acc. diag.

Scale - $1\text{ cm} = 2\text{ m/s}^2$
(diag) (actual)

When the links "OQ" and "the disc" rotate relative to each other at the center of the pin forming a turning pair, the amount of relative rotation is independent of the shapes of the links. Hence the given turning pair can be reduced to a turning pair of OQ and OA, turning relative to each other at O'.

Velocity diagram -

We know that

$$V_{aq} = V_{ao} + V_{oq}$$

1) V_{oq} will be \perp to OQ. Through the pole 'q', draw a line \perp to OQ. Magnitude of $V_{oq} = (3)(2) = 6 \text{ m/s}$.

Take a suitable scale and mark V_{oq} to represent vel. of 'o' relative to 'q'.

2) Now, for the link OA, V_{ao} will be \perp to OA. Magnitude of $V_{ao} = (2)(3) = 6 \text{ m/s}$. Through 'o', draw a line \perp to OA and mark a point 'a' on it such that vector 'oa' represents V_{ao} .

3) From the vel. diagram, $V_{aq} = (4.25)(2) = 8.50 \text{ m/s}$.

Acceleration diagram ; -

No.	Vector	Magnitude	Direction	Sense
1	a_{oq}^c	$= 1(OQ) \omega_{oq}^2 = \frac{V_{oq}^2}{1(OQ)} = \frac{36}{3} = 12 \text{ m/s}^2$	\parallel to OQ	Towards Q in Config-diag.
2	a_{oq}^t	$= 0 \quad \therefore \alpha_{oq} = 0$	—	—
3	a_{ao}^c	$= \frac{V_{ao}^2}{1(AO)} = \frac{36}{2} = 18 \text{ m/s}^2$	\parallel to AO	Towards O. in Config-diag.
4	a_{ao}^t	$= 1(AO) \alpha_{ao} = (2)(6) = 12 \text{ m/s}^2$	\perp to OA	in direction of α_{ao}

$$\begin{aligned} \text{Also, } a_{aq} &= a_{ao} + a_{oq} \\ &= (a_{ao}^c + a_{ao}^t) + (a_{oq}^c + a_{oq}^t) \end{aligned}$$

By setting up the acceleration table, we can draw the acceleration diagram.

From the acceleration diagram,

$$\underline{a_{ag} = 18 \text{ m/s}^2}$$

(along link AO, from A to O)

{ \therefore the direction of a_{ag} is to be applied at point A, while the observer will be at a.



1) (P4) The end A of a bar AB, is constrained to move along the vertical path AD, and the bar passes through a swivel bearing pivoted at C. Draw velocity and acceleration diagrams for the given configuration when A has a velocity of 3 m/s towards D and an acceleration of 25 m/s^2 in the opposite direction.

- Determine -
- 1) The velocity and acceleration of sliding of the bar through the swivel.
 - 2) The angular velocity and angular acceleration of AB.
- (oft 16)(Q.2) (16)

Solution

Let c' be a point on link AB, which is coincident with C.

1) Velocity diagram -

a) V_{ad} is \parallel to AD. Draw V_{ad} . Take a scale of: $\frac{1 \text{ cm (diag)}}{0.5 \text{ m/s (actual)}}$ for the vel. diagram.

b) $V_{c'c}$ will be \parallel to AC' . Through c' , draw a line \parallel to AC' . $V_{c'a}$ will be \perp to AC' . Through a , draw a line \perp to AC' . The intersection of these lines will locate c' .

Now from the velocity diagram,

$$\underline{\text{Velocity of sliding of the bar through the swivel}} \\ = V_{c'c} = 1.55 \text{ m/s.}$$

Angular velocity of link AB

$$= \omega_{ab} = \frac{V_{ab}}{l(AB)} = \frac{V_{ac'}}{l(AC')} = \frac{2.58}{0.465} = 5.55 \text{ rad/s} \quad (\text{anticlockwise})$$

Acceleration diagram

Sr. No.	Vectors	Magnitude	Direction	Sense
1	a_{ad} or a_{ac}	25 m/s^2	to AD	Towards A in config. diag.
2	a_{ca}^c	$= \frac{V_{ca}^2}{r(c'A)} = \frac{(2.58)^2}{0.465} = 14.32 \text{ m/s}^2$	to Ac'	Towards A in config. diag.
3	a_{ca}^t	$= r(c'A) \omega_{c'A} = 4.0 \text{ m/s}^2$	Opposite ⊥ to a_{ca}^c	—
4	a_{cc}^c (Coriolis)	$= 2 V_{cc'} \omega_{c'A}$ $= (2)(1.55)(5.55)$ $= 17.21 \text{ m/s}^2$	opp. to that of a_{ca}^c is. Opp. to that of Cor. comp of a_{ca}^c of block	—
5	a_{cc} (sliding)	$= 27.6 \text{ m/s}^2$	⊥ to a_{ca}^t	—

a) The magnitude and direction of acceleration of A is known. Hence a_{ac} can be drawn.

b) a_{ca}^c is || to Ac' and towards A. Hence (from c')

a_{ca}^c can be drawn.

c) a_{ca}^t can not be drawn since its magnitude is not known. However, it is ⊥ to $c'A$. Hence through 'c', a line ⊥ to $c'A$ is drawn. (line 1)

d) The block C is moving towards A, and Ac' is rotating about A in anticlockwise direction. Hence (This can be made out from the direction of $V_{c'c}$. $V_{c'c}$ will be in opposite direction. Hence as the link moves in direction of $V_{c'c}$, the block will be moving relative to link in the direction $V_{c'c}$)

Hence, the direction of Coriolis component of $a_{cc'}$ will be as shown in fig. (b)

Velocity of block relative to link = $v_{cc'}$

Angular velocity of link = $\omega_{c'a}$

\therefore Coriolis comp. of acceleration of block relative to the link = $a_{cc'} = (2v_{cc'}\omega_{c'a}) = (2)(1.55)(5.55) = \frac{17.21}{5} \text{ m/s}^2$

This will be the tangential acceleration of C relative to c' . The tangential acceleration of c' relative to C will be therefore in the opposite direction of $a_{cc'}$. ~~Through~~

Hence through 'c', a line // to the direction of $a_{cc'}$ is drawn, and a point ' c'_c ' is obtained.

e) The sliding acceleration of c' relative to C must be \perp to $a_{c'c}^t$. Hence a line \perp to $a_{c'c}^t$ is drawn through ' c'_c '. (line 2)

The intersection of line 1 and line 2 locate point ' c' '.

From the acceleration diagram,
acceleration of sliding of bar through swivel

$$= a_{c'c}^{\text{sliding}}$$

$$= 27.6 \text{ m/s}^2$$

Angular acceleration of AB

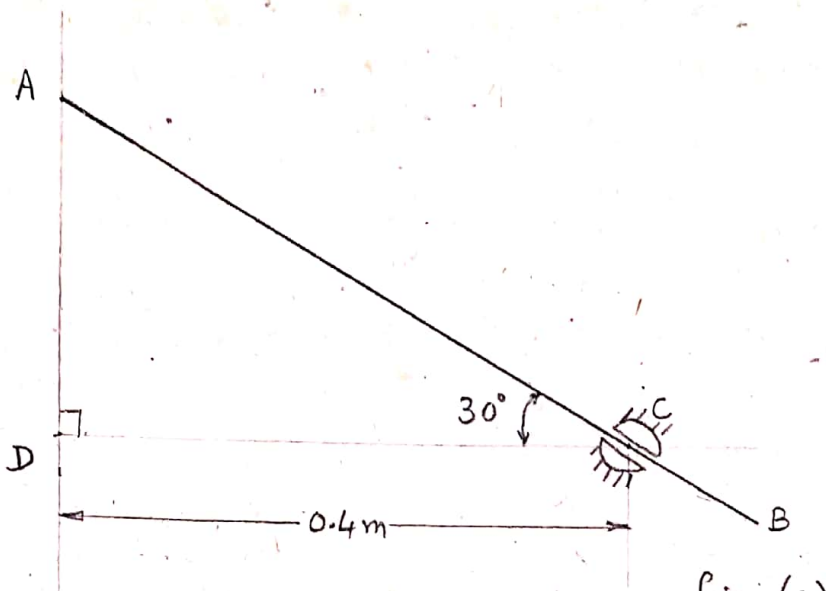
$$= \alpha_{ab}$$

$$= \alpha_{c'a}$$

$$= \frac{a_{c'a}^t}{l(C'A)} = \frac{4.0}{0.465} = 8.6 \text{ rad/s}^2$$

(Relative to A, the point c' will be having ang. accⁿ in the direction sense given by $a_{c'a}^t$)

(P4)



Scale - 1 cm = 0.05m (diag.) (actual)

fig.(a)

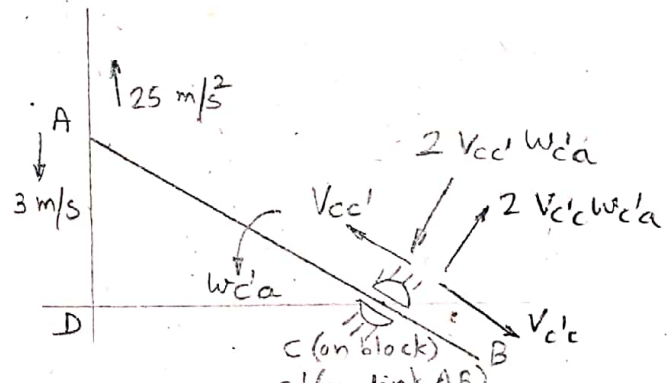
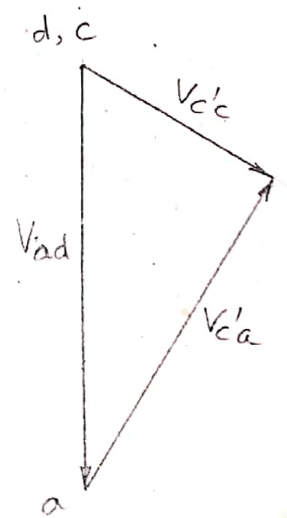
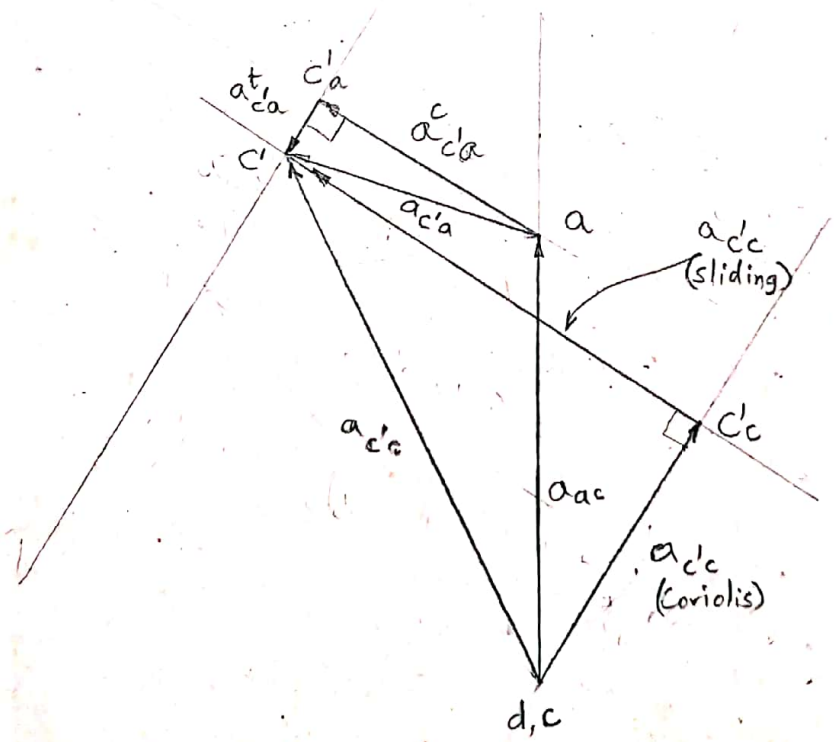


fig.(b)



vel. diagram

Scale - 1 cm = 0.5 m/s (diag.) (actual)



Accn. diag.

Scale - 1 cm = 4 m/s^2 (diag.) (actual)

- 4) (P5) In the figure shown, a rod PR is constrained by guides to move horizontally and is driven by a crank OA and a sliding block at P. For the given configuration, determine graphically or otherwise, the acceleration of PR when OA has an angular velocity of 5 rad/s in counter clockwise direction and an angular acceleration of -35 rad/s^2 i.e. clockwise. Also find the acceleration of sliding of the link AO through the sliding block P.

Solution -

Assume point P' on link OA, and coincident with P.

Velocity diagram -

1) $V_{P'O} = r(P'O)(\omega_{P'O}) = (3.425)(5) = 17.13 \text{ m/s}$.

$V_{P'O}$ will be \perp to $P'O$. Taking a scale of 1 cm (diag.) = 4 m/s (actual), draw $V_{P'O}$ in proper direction.

2) ~~$V_{P'O}$ will be \perp to $P'O$. Through~~
 $V_{PP'}$ will be \parallel to $P'O$. Through 'p', draw a line \parallel to $P'O$. V_{PO} will be \parallel to PR. Through 'o', draw a line \parallel to PR. The intersection of these two lines locates point P.

Thus, the velocity diagram is completed.

Acceleration diagram -

Sr. No.	Vector	Magnitude	Direction	Sense
1	$a_{p/o}^c$	$= \frac{V_{p/o}^2}{r(p/o)} = \frac{(17.13)^2}{3.425} = 85.68 \text{ m/s}^2$	// to p/o	Towards O in config. diag.
2	$a_{p/o}^t$	$= \omega(p/o) \times r_{p/o} = (3.425)(35) = 119.88 \text{ m/s}^2$	\perp to $a_{p/o}^c$	in the direction of $\omega_{p/o}$
3	$a_{pp'}^{\text{(sliding)}}$	$= 73.5 \text{ m/s}^2$	\perp to $a_{pp'}^t$	in the direction of $\omega_{pp'}$
4	$a_{pp'}^{\text{(Coriolis)}}$	$= 2 V_{pp'} \omega_{op'} = 2(10)(5) = 2(10)(5) = 100 \text{ m/s}^2$	\perp to OP	As shown in config. diag. by $(2 V_{pp'} \omega_{op'})$
5	a_{po}	$= 23.25 \text{ m/s}^2$	// to PR	—

1) Draw $a_{p/o}^c$ and $a_{p/o}^t$.

2) The direction of $a_{pp'}$ is known. It is parallel to OP.

First draw $a_{pp'}^t$, because its direction and magnitude are known. The direction of this Coriolis component will be as shown in the configuration diagram. Hence draw $a_{pp'}^c$. $a_{pp'}^c$ will be \perp to $a_{pp'}^t$. Hence through p' , draw a line \perp to $a_{pp'}^t$. (line 1)

(15)

3) a_{p0} will be \parallel to PR, because P is constrained to move in horizontal direction. Hence through 'o', draw a line \parallel to PR. (line 2)

The intersection of line 1 and line 2 locates point 'P'.

From the accⁿ diagram,

① Acceleration of PR

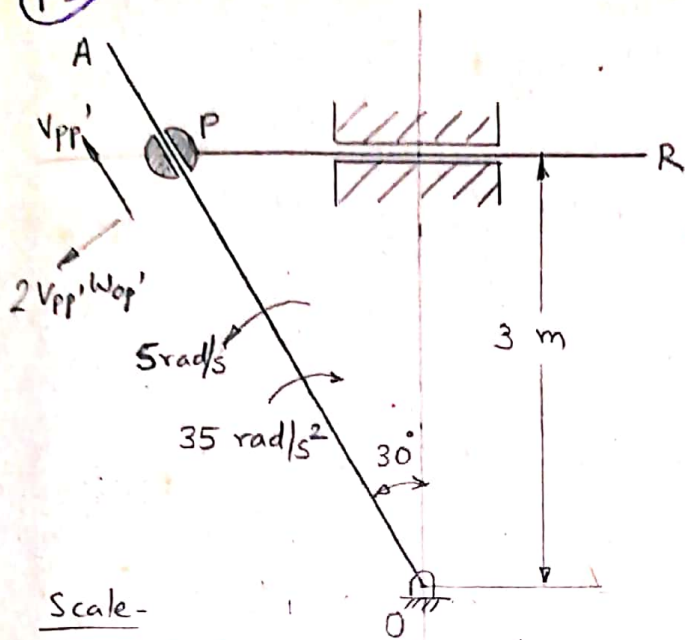
$$= a_{p0}$$

$$= 23.25 \text{ m/s}^2$$

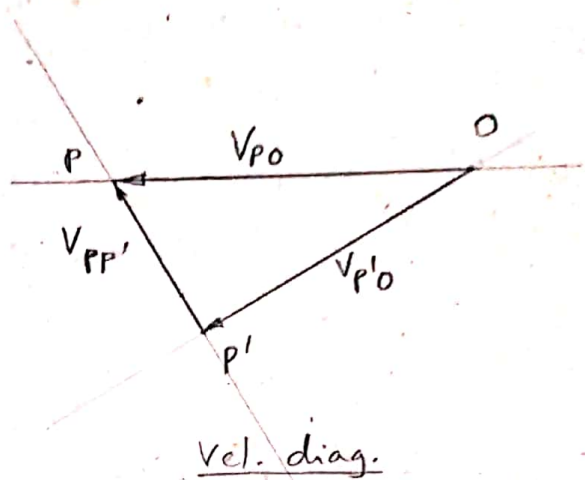
② Acceleration of sliding of the link AO through the sliding block P

$$= a_{pp'}(\text{sliding}) = 73.5 \text{ m/s}^2$$

(P5)



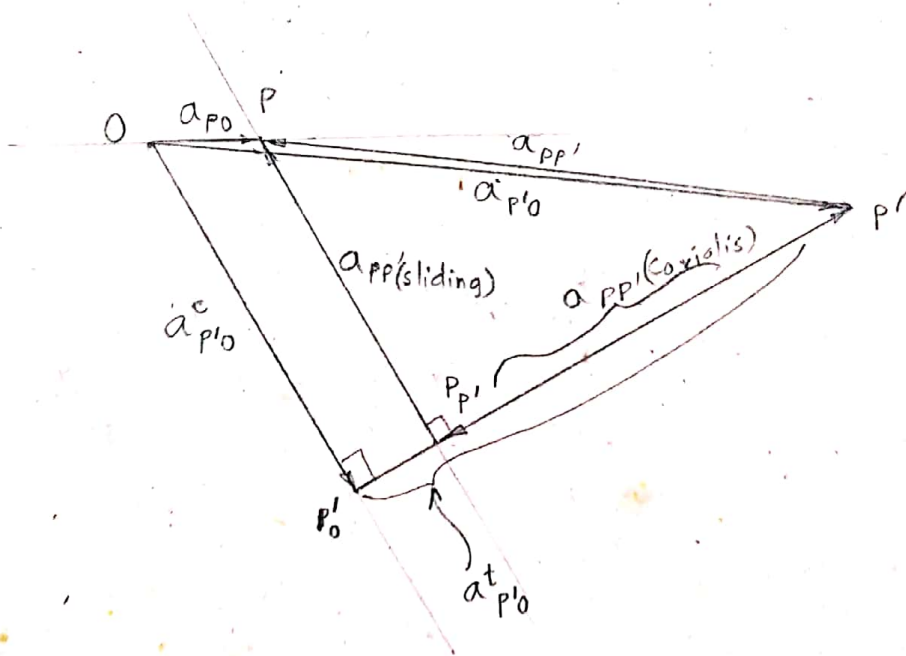
Scale -
 1 cm = 0.5 m
 (diag.) (actual)



Scale
 1 cm = 4 m/s
 (diag.) (actual)



PR is a link which forms a turning pair with the block, at point P.



Accn. diag.
 Scale - 1 cm = 15 m/s^2
 (diag.) (actual)

① P14 In the figure, a bent lever CBD is pivoted at the fixed point C. The angle CBD is 90° . The crank OA rotates anticlockwise at 180 rpm about the fixed point O. The crank pin A slides in a closely fitting slot in the lever, as shown. For the given position, in which the crank is at an angle of 45° from the horizontal, find the angular velocity and the angular acceleration of the lever and also the acceleration of point B.

Solution

Velocity diagram -

1) $V_{AO} = r(AO) \left(\frac{2\pi(180)}{60} \right) = (0.05) \left(\frac{2\pi(180)}{60} \right) = 0.94 \text{ m/s}$

Draw V_{AO} .

2) Assume A_1 to be a point on link CBD, but coincident with A.

V_{A_1A} will be \parallel to BD. Through 'a', draw a line \parallel to BD.

V_{A_1C} will be \perp to AC. Through 'c', draw a line \perp to AC. The intersection of these lines locates 'a'.

3) Locate point 'b' by building a velocity image of link CBD; in the velocity diagram.

Actual $r(AC)$ on the config. diagram = 4.65 cm.
 $r(a_1c)$ on the accⁿ diagram = 1.4 cm.

$\therefore 4.65 \text{ cm} \longrightarrow 1.4 \text{ cm}$

$\frac{2 \text{ cm}}{\text{(for BC on config. diag.)}} \longrightarrow \frac{2 \times 1.4}{4.65} = 0.6 \text{ cm}$

$\frac{4.15 \text{ cm}}{\text{(for A, B on config. diag.)}} \longrightarrow \frac{4.15 \times 1.4}{4.65} = 1.25 \text{ cm}$

Thus, point 'b' can be located.

4) Now extend 'ba' to point 'd' (if desired.)

From velocity diagram,

1) Angular velocity of lever CBD

$$= \frac{V_{bc}}{l(BC)}$$

$$\therefore \omega_{cbd} = \frac{0.21}{(0.05)} = 4.2 \text{ rad/s} \curvearrowright$$

Acceleration diagram -

Sr. no.	Vector	magnitude	Direction	Sense
1	a_{ao}^c	$= \frac{V_{ao}^2}{l(AO)} = \frac{(0.94)^2}{(0.05)} = 17.67 \text{ m/s}^2$	to AO	Towards O in fig.
2	a_{ao}^t	$= 0 \quad \because \alpha_{ao} = 0$	—	—
3	a_{aia}^{cor}	$= 2(V_{aia})\omega_{cbd} = 2(0.65)(4.2) = 5.46 \text{ m/s}^2$	As shown in fig. (⊥ to BD)	—
4	a_{aia}^s	$= 5.05 \text{ m/s}^2$	⊥ to a_{aia}^{cor}	—
5	a_{aic}^c	$= \frac{V_{aic}^2}{l(Aic)} = \frac{(0.48)^2}{(0.116)} = 1.99 \text{ m/s}^2$	to Aic	Towards C in fig.
6	a_{aic}^t	$= l(Aic)\alpha_{aic} = 24.5 \text{ m/s}^2$ $\therefore \alpha_{aic} = \frac{24.5}{0.116} = 211.21 \text{ rad/s}^2$	⊥ to a_{aic}^c	—
7	a_{bc}^c	$= \frac{V_{bc}^2}{l(BC)} = \frac{(0.21)^2}{(0.05)} = 0.88 \text{ m/s}^2$	to BC	Towards C in fig.
8	a_{bc}^t	$= l(BC)\alpha_{bc} = 206 \text{ m/s}^2$ $\therefore \alpha_{bc} = \frac{206}{0.05} = 4120 \text{ rad/s}^2$	⊥ to a_{bc}^c	—
9	a_{ba1}^c	$= \frac{V_{ba1}^2}{l(BA1)} = \frac{(0.42)^2}{(0.104)} = 1.7 \text{ m/s}^2$	to BA1	Towards A1 in fig.
10	a_{ba1}^t	$= l(BA1)\alpha_{ba1} = 212.5 \text{ m/s}^2$ $\therefore \alpha_{ba1} = \frac{212.5}{0.104} = 2043.27 \text{ rad/s}^2$	⊥ to a_{ba1}^c	—

1) a_{ao} is drawn.

2) a_{aia}^{cor} is drawn. a_{aia}^s will be ⊥ to a_{aia}^{cor} . Through 'aia', a line ⊥ to a_{aia}^{cor} is drawn. (line 1)

3) a_{aic}^c is drawn. a_{aic}^t will be ⊥ to a_{aic}^c . Through 'aic', a line ⊥ to a_{aic}^c is drawn. (line 2)

2)

The intersection of these two locates 'a'.

4) 'b' can be located by building accⁿ image of ΔABC on the accⁿ diagram.

or

a_{bc}^c is drawn. a_{bc}^t will be \perp to a_{bc}^c . Through 'b', a line \perp to a_{bc}^c is drawn. (line 1)

5) a_{ba}^c is drawn. a_{ba}^t will be \perp to a_{ba}^c . Through 'ba', a line \perp to a_{ba}^c is drawn. (line 2)

The intersection of these lines locates 'b'.

From the accⁿ diagram,

1) Ang. accⁿ of lever

$$= \alpha_{bc} = \alpha_{ba} = \alpha_{a,c}$$

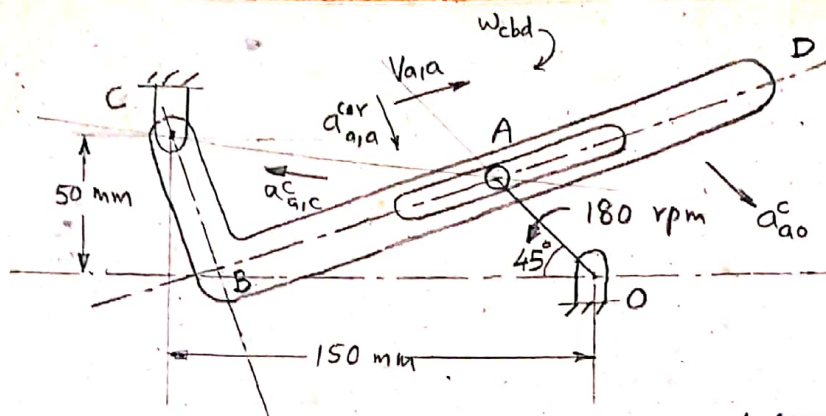
$$\alpha_{bc} = \frac{211.21}{52.5} \text{ rad/s}^2 \quad \curvearrowright$$

2) Accⁿ of point B

$$= a_{bo} = \frac{10.7}{2.58} \text{ m/s}^2$$



P14

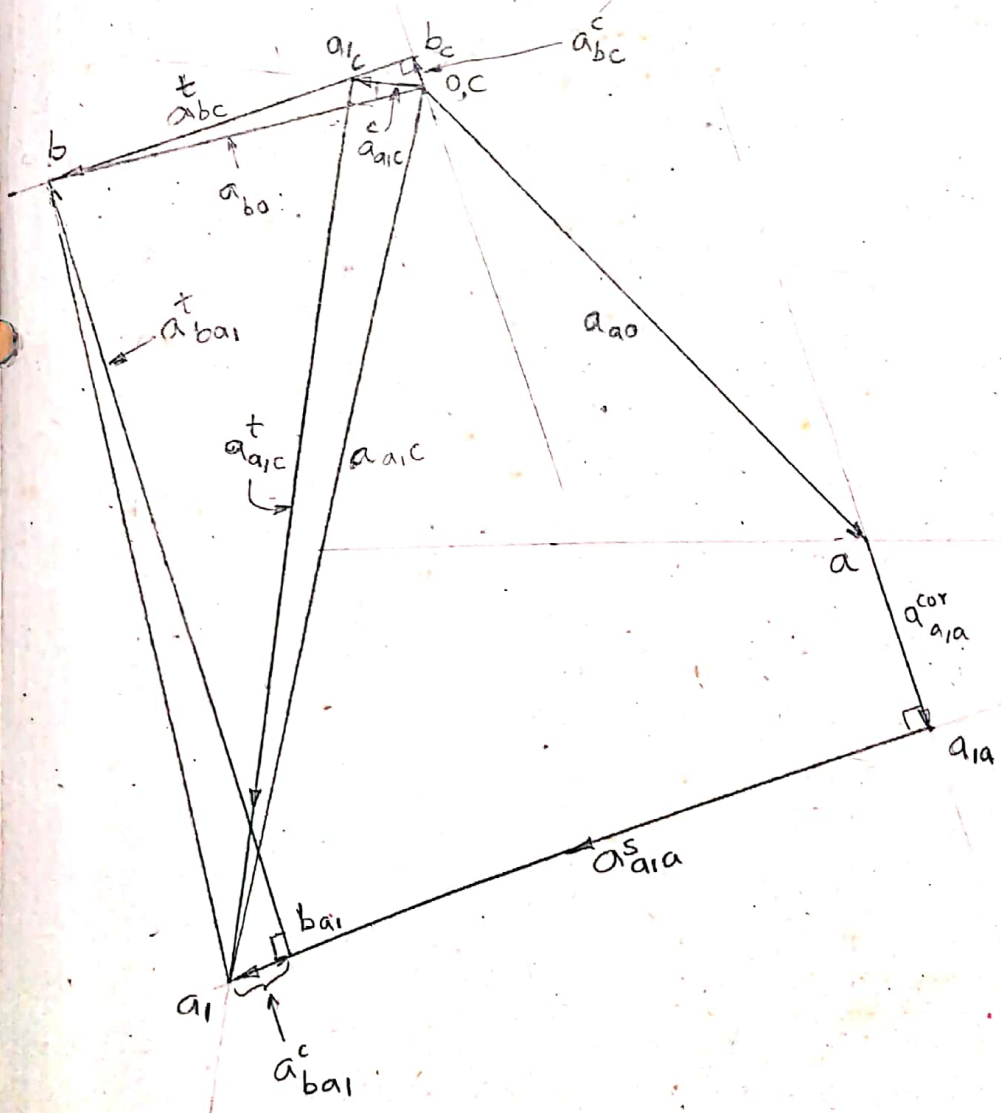
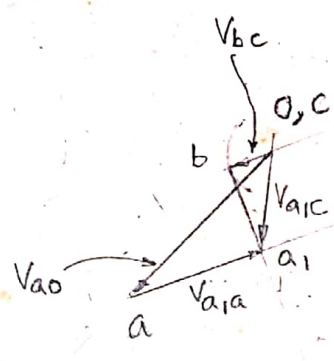


$l(OA) = 50 \text{ mm}$
 $l(BC) = 50 \text{ mm}$

1 cm = 25 mm
 (diag.) (actual)

Vel. diagram

1 cm = 0.333
 (diag.) m/s
 (actual)



Acc. diagram

1 cm = 2 m/s²
 (diag.) (actual)